

## 10) FURTHER GEOMETRY OF THE CONIC SECTIONS (EXCELLENCE)

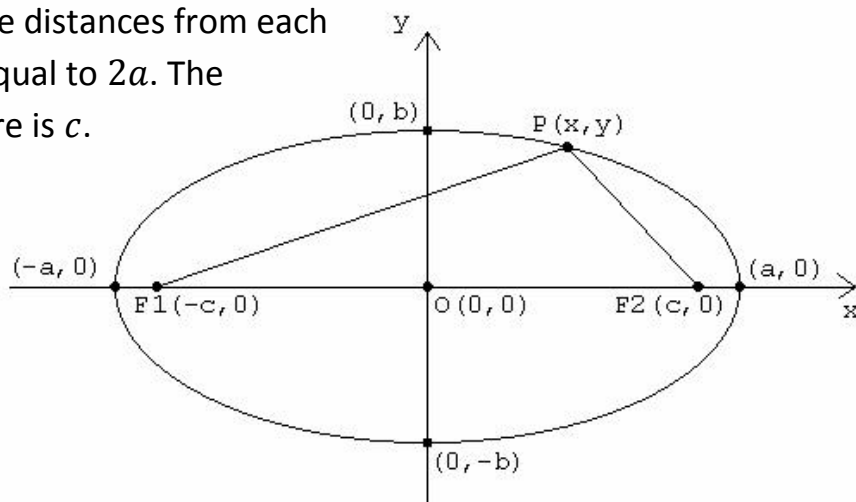
We shall now consider conic sections as **paths** or **loci** (i.e. the **set of points** that follow given rules). The important geometric features of conic sections are their **focus/foci**, **eccentricity**, and the **equations** that relate to the focus/foci.

It is important to note that the constants  $a$  and  $c$  are used to represent either the centre of the conic section, or the focus of the conic section, depending on the conic.

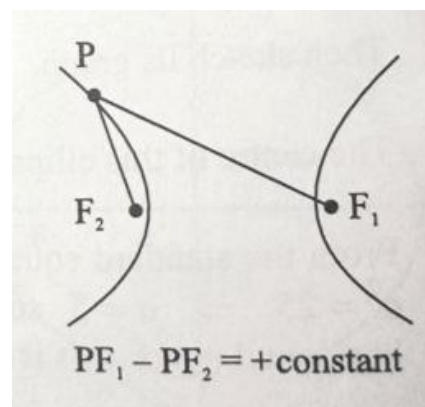
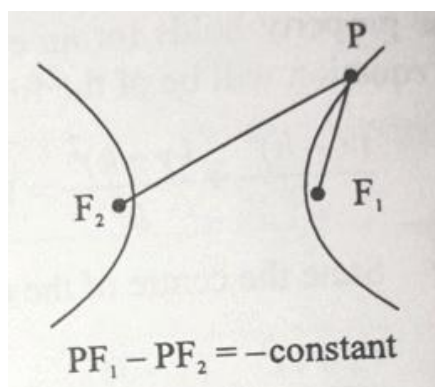
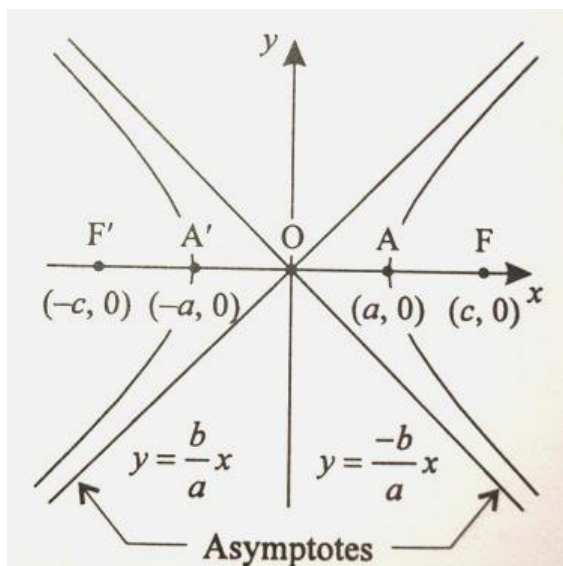
### FOCUS

The foci (plural of focus) are a pair of special points which help to describe a particular conic section. The foci always lie on the horizontal line of symmetry of the conic sections.

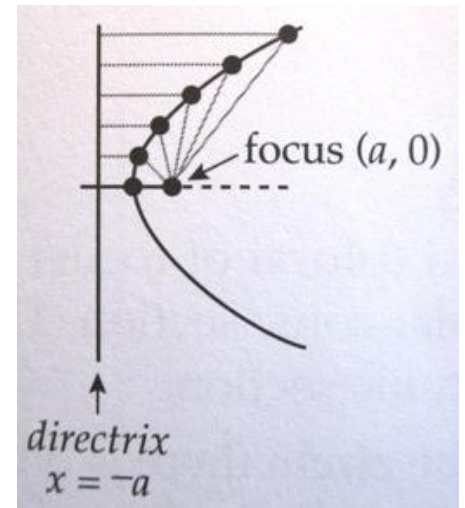
- **Circle:** In the circle there is only one focus, which is the centre of the circle. All the points of the circle are **equidistant** from the focus.
- **Ellipse:** In the ellipse, the *sum* of the distances from each of the two foci is constant, and is equal to  $2a$ . The distance of each foci from the centre is  $c$ .



- **Hyperbola:** In a hyperbola, the *difference* of the distances from each of the two foci is constant and is equal to  $2a$ . The distance of each foci from the centre is  $c$ .



- **Parabola:** In a parabola, there is only one focus. There is a line called the **directrix**; and for any point on the parabola, the distance from the focus is the same as the horizontal distance from the directrix. In other words, the path the parabola forms is always the same distance from a straight line (the directrix) as it is from a fixed point (the focus).



## ECCENTRICITY

Eccentricity is a measure of how much a shape deviates from a perfect circular shape.

- Circles have an eccentricity of 0.
- Ellipses can go from almost circular (eccentricity close to 0) to just short of breaking down as a closed shape (eccentricity close to 1)
- Parabolas are not closed shapes, and their eccentricity is 1.
- Hyperbolae have two branches, and their eccentricity is greater than 1. As  $e$  increases the asymptotes steepen and the hyperbola widens.

## EQUATIONS FOR FEATURES OF CONIC SECTIONS

### **Parabola:**

Eccentricity,  $e = 1$

Focus is at  $(a, 0)$

Directrix is  $x = -a$

### **Ellipse:**

Eccentricity,  $e = \frac{c}{a}$

Foci are at  $(c, 0)$  and  $(-c, 0)$

$$c^2 = a^2 - b^2$$

### **Hyperbola:**

Eccentricity,  $e = \frac{c}{a}$

Foci are at  $(c, 0)$  and  $(-c, 0)$

$$c^2 = a^2 + b^2$$